TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 5 2016

Calculator Free
Matrices

STUDENT'S NAME ____

DATE: **TIME:** 50 minutes **MARKS**: 55

INSTRUCTIONS:
Standard Items: Pens, pencils, ruler, eraser.
Special Items: Formula sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

Determine matrices A and B given

- $A + B = \begin{bmatrix} 9 & -1 & 4 \\ 6 & -5\cdot5 & 3\cdot5 \end{bmatrix}$
- $a_{23} = b_{22} = 0\cdot5$
- $a_{21} = b_{12} = 2$
- $a_{11} = b_{11} - 1 = b_{13} + 1$

2. (6 marks)

(a) $A = \begin{bmatrix} 6k & k-7 \\ 3k & k+2 \end{bmatrix}$. Determine the value(s) of k such that A is singular. [3]

(b) Prove that $(PQ)^3 = I$, given that $QPQ = P^{-1}Q^{-1}P^{-1}$ and I is the identity matrix. [3]

3. (8 marks)

Two matrices A and B are related by the equation $A + B = AB$.

(a) What does this equation imply about the dimensions of A and B? [2]

(b) (i) Use the equation given above to prove that $(I - A)(I - B) = I$ where I denotes an appropriate identity matrix. [3]

(ii) Hence determine the inverse matrix $(I - A)^{-1}$ when

$$B = \begin{bmatrix} 8 & -8 & 5 \\ -4 & 6 & -3 \\ 1 & -1 & 2 \end{bmatrix}$$ [3]

4. (8 marks)

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}.$$

There are only six possible matrices that can result from calculating A^n where n = 1, 2, 3, 4, …

(a) Determine the six possible matrices [5]

(b) Show why there can be no more than these six. [1]

(c) Using only this information, and showing working, determine A^{21}. [2]

5. (7 marks)

$$M = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

(a) Determine M^2 [1]

(b) Determine the image of the point (8, 0) under the transformation represented by M^{99} [2]

(c) Determine the coordinates of the point whose image is (0, 10) under the transformation represented by M^{99}. [2]

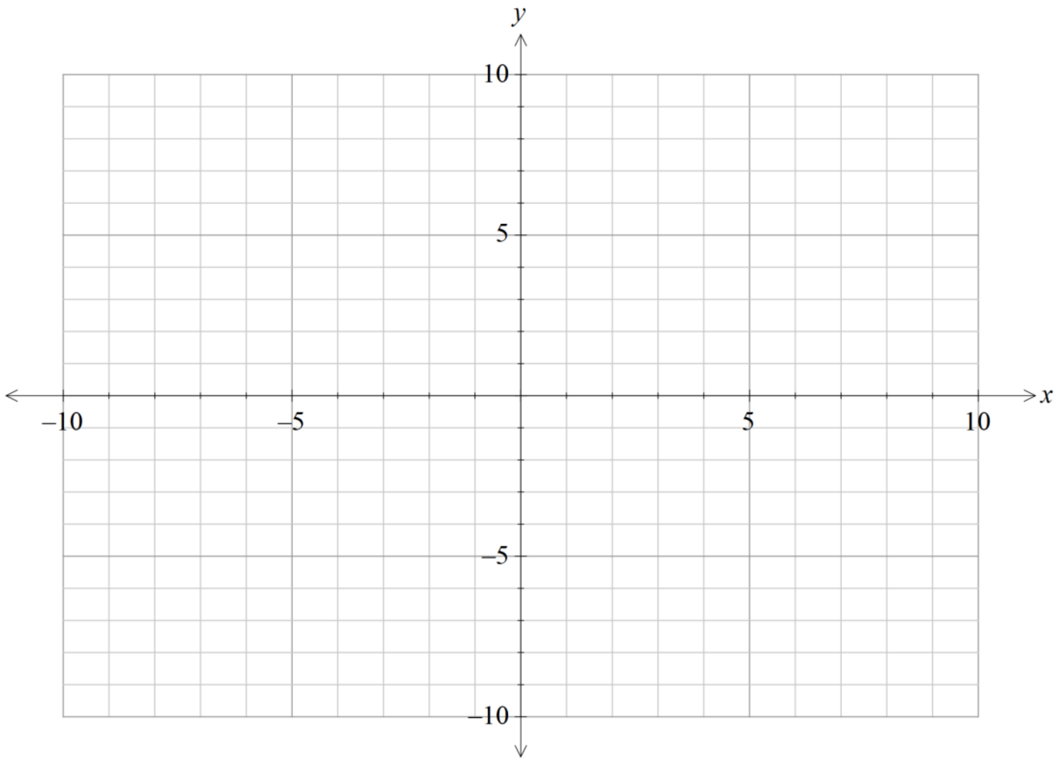
(d) Comment on the geometrical effects of the transformation represented by M^{99}. [2]

6. (15 marks)

A parallelogram formed by the points $A(3, -1)$, $B(4, 2)$, $C(-1, 3)$ and $D(-2, 0)$ is transformed into $A'B'C'D'$ by the matrix $\begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$.

(a) What are the coordinates of A', B', C' and D'. [2]

(b) Draw $ABCD$ and $A'B'C'D'$ on the axes below. [4]



(c) Compare the area of ABCD and $A'B'C'D'$. [1]

(d) Transform $A'B'C'D'$ to $A''B''C''D''$ using the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and draw the new quadrilateral on your axes. [3]

(e) Describe, in words, the transformation of ABCD to $A''B''C''D''$. [2]

(f) What single matrix would transform $A''B''C''D''$ back to $ABCD$? [3]

7. (7 marks)

(a) Given that matrices A and B are commutative for multiplication, simplify the following expression. Justify your answer. [3]

$$A^2BA^{-1}B^{-1}$$

(b) Let W be an $n \times n$ non-singular matrix such that $6W^2 - 2W + I = 0$ where I is the identity matrix and O is the zero matrix.
Determine p and q such that $W^{-1} = pW^2 + qI$. [4]